OCR Mechanics 4 June 2003

1 $\omega_1^{\,2} = \omega_0^{\,2} + 2\dot{\omega}\theta$ (equivalent of the linear $v^2 = u^2 + 2as$)

$25^2 = 15^2 + 2 \times \dot{\omega} \times 160$

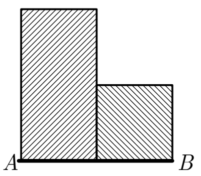
$\dot{\omega} = \mathbf{1 \cdot 25}$ rad s^{-1}

[2]

$t = \dfrac{\omega_1 - \omega_0}{\dot{\omega}} = \dfrac{25-15}{1 \cdot 25} = \mathbf{8}$ s

[2]

2



$I = \frac{4}{3}(2m)a^2 + \frac{4}{3}m\left(\frac{1}{2}a\right)^2 = \frac{4}{3}ma^2\left(2+\frac{1}{4}\right) = \frac{4}{3}ma^2 \times \frac{9}{4} = \mathbf{3ma^2}$ (show)

[3]

perpendicular axes rule …

$I_A = I_{AB} + I_{AF} = 3ma^2 + 3ma^2 = \mathbf{6ma^2}$

[2]

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3 energy considerations …

K.E. gained = loss in G.P.E. $-$ work done by friction

$\frac{1}{2}I\omega^2 = mgh - C\theta$

$\frac{1}{2}\cdot\left(\frac{4}{3}\times 0\cdot 75 \times 0\cdot 8^2\right)3^2 = 0\cdot 75 \times 9\cdot 8 \times 0\cdot 8 - C \times \frac{\pi}{2}$

$C = \frac{6}{\pi} = 1\cdot 90985... = \mathbf{1 \cdot 91\ Nm}$

[4]

conservation of angular momentum …

$(0\cdot 56 + 0\cdot 64)\omega = 0\cdot 56 \times 4\cdot 2 \ + \ 0\cdot 64 \times {}^{-}3$

$\boldsymbol{w = 0 \cdot 36}$ **rad s^{-1}**

[3]

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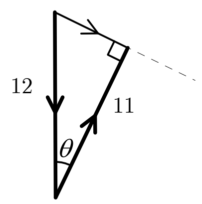
4 For B to get as close as possible to C, then ${}_B\mathbf{v}_C$ must be as close to due east as possible …

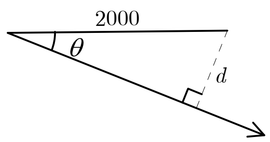
${}_B\mathbf{v}_C = {}_B\mathbf{v}_G - {}_C\mathbf{v}_G$



bearing on which B must sail $= \cos^{-1}\left(11/12\right)$

$= 23\cdot 5564... = \mathbf{023 \cdot 6^\circ}$

[4]



minimum separation between B and C $= 2000\sin\theta$

$= 2000\sqrt{1-\left(\frac{11}{12}\right)^2}$

$= 799\cdot 305...$

$= \mathbf{799\ m}$ (3 s.f.)

[4]

5

$$\text{mass} = 350\int_0^8 \pi x^{\frac{2}{3}}\,\mathrm{d}x = 350\pi\left[\tfrac{3}{5}x^{\frac{5}{3}}\right]_0^8 = 210\pi \times 32 = \mathbf{6720\pi} \qquad \text{(show)}$$

[3]

mass of 'elemental disc' $= \rho\pi y^2\delta x = 350\pi x^{\frac{2}{3}}\delta x = 350\pi x^{\frac{2}{3}}\delta x$

$$6720\pi\bar{x} = \int_0^8 x\left(350\pi x^{\frac{2}{3}}\right)\mathrm{d}x = 350\pi\left[\tfrac{3}{8}x^{\frac{8}{3}}\right]_0^8 = \tfrac{525}{4}\pi \times 256 \qquad\qquad \boldsymbol{\bar{x}} = \tfrac{33600}{6720} = \mathbf{5}$$

[3]

M.o.I. of 'elemental disc' $= \frac{1}{2}mr^2 = \frac{1}{2}\left(350\pi x^{\frac{2}{3}}\delta x\right)x^{\frac{2}{3}} = 175\pi x^{\frac{4}{3}}\delta x$

$$I = \int_0^8 175\pi x^{\frac{4}{3}}\,\mathrm{d}x = 175\pi\left[\tfrac{3}{7}x^{\frac{7}{3}}\right]_0^8 = 75\pi \times 128 = \mathbf{9600\pi \ \ kg\,m^2}$$

[4]

6

$I = \frac{1}{2}mr^2 + Mr^2 = \frac{1}{2}\times 0\cdot 08 \times 0\cdot 35^2 \ + \ 0\cdot 24 \times 0\cdot 35^2 = \mathbf{0\cdot 0343 \ \ kg\,m^2}$

[3]

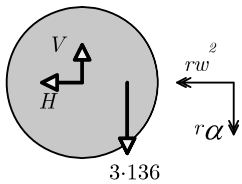
$0\cdot 32\bar{x} = 0\cdot 08 \times 0 + 0\cdot 24 \times 0\cdot 35 \qquad\qquad \boldsymbol{\bar{x} = 0\cdot 2625}$

[2]

when OP is horizontal …

$$C = I\alpha$$
$$0\cdot 32 \times 9\cdot 8 \times 0\cdot 2625 = 0\cdot 0343\alpha$$
$$\boldsymbol{\alpha = 24 \ \ \mathbf{rad\,s^{-2}}}$$

[2]



N2$(\leftarrow)$ $\qquad H = m\left(r\omega^2\right) = 0\cdot 32 \times 0\cdot 2625 \times 5^2 = 2\cdot 1$

N2$(\downarrow)$ $\qquad 3\cdot 136 - V = m(r\alpha)$

$\qquad V = 3\cdot 136 - 0\cdot 32 \times 0\cdot 2625 \times 24 = 1\cdot 12$

hence the m**agnitude of the force** $= \sqrt{2\cdot 1^2 + 1\cdot 12^2} = \mathbf{2\cdot 38 \ \ N}$

[6]

7 Potential Energy Function

$$V = \text{G.P.E. of rod} \; + \; \text{E.P.E. of } R_1B \; + \; \text{E.P.E. of } R_2B$$

$$= mg\,(a\cos\theta) + \tfrac{1}{2}\cdot\tfrac{\frac{1}{4}mg}{a}\left\{(2a+2a\sin\theta)^2 + (2a-2a\sin\theta)^2\right\}$$

$$= mga\cos\theta + \tfrac{1}{2}mga\left(2+2\sin^2\theta\right)$$

$$= \boldsymbol{mga\left(1+\cos\theta+\sin^2\theta\right)} \qquad \text{(show)}$$

[5]

conservation of mechanical energy …

$$mga\left(1+\cos\theta+\sin^2\theta\right) + \tfrac{1}{2}\left(\tfrac{4}{3}ma^2\right)\dot{\theta}^2 = \text{constant}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right) \qquad mga\,\dot{\theta}\,(2\sin\theta\cos\theta - \sin\theta) + \tfrac{4}{3}mga\,\dot{\theta}\,\ddot{\theta} = 0$$

$$\ddot{\theta} = {}^{-}\left(\frac{3g}{4a}\right)\sin\theta\,(2\cos\theta - 1)$$

for small oscillations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

$$\ddot{\theta} \approx {}^{-}\left(\frac{3g}{4a}\right)\theta$$

and so we have approximate SHM with period $T = 2\pi\sqrt{\dfrac{4a}{3g}} = \boldsymbol{4\pi\sqrt{\dfrac{a}{3g}}}$

[8]